

BOOK REVIEW

Spectra and Pseudospectra. By LLOYD N. TREFETHEN & MARK EMBREE. Princeton University Press, 2005. 606 pp. ISBN 0 691 11946 5.

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Matrices are more than the set of their eigenvalues. Linear operators are more than their spectrum. Some linear systems that spectral analysis would call stable exhibit large transient growth that can trigger nonlinear effects. This phenomenon is intimately tied to the idea of ‘pseudospectra’ – the sets in the complex plane that are, in a relevant norm, close to the spectrum of the linear system in question. The authors of this evangelical book have set out to explain, in 60 short chapters, the mathematics, the computation, and the applications of pseudospectra.

Six of the chapters are an introduction to the ideas of pseudospectra, six of them are on the computation of pseudospectra, and the remaining 48 chapters show the applications of pseudospectra in various different fields and to various different problems. Some chapters are accessible to a sufficiently determined (mathematics) undergraduate; others are rather more specialist. It is just about possible to dip into the book at random, and the authors claim that they do not expect any reader to read the whole book from cover to cover.

In places the book is really superb. Chapters 14–19 – on transient behaviour in dynamical systems – are fascinating, with enough examples to convince even the sceptic of the relevance of these ideas. Chapters 7–9 – on Toeplitz matrices and operators – and chapters 35–38 – on random matrices – are also enjoyable (and, at least to me, of more fluid-mechanical interest than some of the ‘fluid-mechanical’ problems discussed elsewhere in the book). Chapters 56 and 57 – on transient behaviour in Markov chains and in models of card shuffling – are both wonderful fun (and also show that some interesting pseudospectral problems have nothing to do with Hilbert spaces). Chapters 39–44 – on the computation of pseudospectra – are, of course, definitive.

However, the book also has its bad points. The four chapters of the book specifically on fluid mechanics are among the least satisfying. The authors do not really move much beyond linear stability problems in simple channel and pipe flows. Whilst a discussion of the Orr–Sommerfeld operator (chapter 22) is certainly merited in any book on pseudospectra, much of the rest of these chapters would perhaps be better as part of the general discussion of transient behaviour in dynamical systems. If your interest in linear systems is confined to the linearized Navier–Stokes equations, you will find more meat in Schmid & Henningson (2001).

Having 60 short chapters seems, in principle, a good idea. However, in many cases I would have liked to see much more material on some of the topics; there is a tendency for the book to refer the reader to the literature just as things start getting interesting. I would have preferred to see half as many topics treated, but in twice as much detail. It also seems that some of the chapters have just been put in to make up the numbers. Chapter 54 – on group velocity – is entirely content-free, and is a particular irritant. An analogy is claimed between pseudospectra and group velocity that seems to consist of little more than the fact that they are both properties of

linear systems that have something to do with the temporal behaviour of the linear system, and have a little to do with eigenvalues.

The authors have set out to write both an introduction and a definitive guide to pseudospectra. A love of the subject and a fascination with its ideas shines through the writing, which is clear throughout. The book covers a broad selection of topics and applications, and I suspect that most applied mathematicians will find something to interest them in it.

REFERENCE

SCHMID, P. J. & HENNINGSON, D. S. 2001 *Stability and Transition in Shear Flows*. Springer.

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